Pascal's Triangle

It will be useful to remember Pascal's Triangle, or at least remember how to generate the triangle. The first 7 rows of the triangle are written below.

							1							
						1		1						
					1		2		1					
				1		3		3		1				
			1		4		6		4		1			
		1		5		10		10		5		1		
	1		6		15		20		15		6		1	
1		7		21		35		35		21		7		1

Expanding a bracket

The Binomial expansion is used at this level to expand brackets in the form $(x + a)^n$. A table of expansions are shown below:

$(x + a)^0$	$1(x)^0(a)^0$
$(x + a)^1$	$1(x)^1(a)^0 + 1(x)^0(a)^1$
$(x + a)^2$	$1(x)^2(a)^0 + 2(x)^1(a)^1 + 1(x)^0(a)^2$
$(x + a)^3$	$1(x)^3(a)^0 + 3(x)^2(a)^1 + 3(x)^1(a)^2 + 1(x)^0(a)^3$
$(x + a)^4$	$1(x)^4(a)^0 + 4(x)^3(a)^1 + 6(x)^2(a)^2 + 4(x)^1(a)^3 + 1(x)^0(a)^4$
$(x + a)^5$	$1(x)^5(a)^0 + 5(x)^4(a)^1 + 10(x)^3(a)^2 + 10(x)^2(a)^3 + 4(x)^1(a)^4 + 1(x)^0(a)^5$
$(x + a)^6$	$1(x)^6(a)^0 + 6(x)^5(a)^1 + 15(x)^4(a)^2 + 20(x)^3(a)^3 + 15(x)^2(a)^4 + 6(x)^1(a)^5 + 1(x)^0(a)^6$
$(x + a)^7$	$1(x)^7(a)^0 + 7(x)^6(a)^1 + 20(x)^5(a)^2 + 35(x)^4(a)^3 + 35(x)^3(a)^4 + 21(x)^2(a)^5 + 7(x)^1(a)^6 + 1(x)^0(a)^7$

You can note the patterns building up in the table, and how number from Pascal's triangle is used as the *coefficient* – that is the number that appears before the letters.

The combination function ${}^{n}C_{r}$

The other method for finding the first number of the of each of these sequences is to use the *combination* function on the GDC.

Casio

Finding ${}^{8}C_{5}$
In RUN mode
OPTN, F6, F3 PROB
8, F3 (nCr), 5, EXE

Answer = 56

TI

Finding ${}^{8}C_{5}$
8, MATH, Tab across to PRB,
choose 3:nCr ENTER 5

Answer = 56

Guided example

Find the 5th term in the expansion of $(3x - 2)^7$.

Answer

As the expansion is to the power of 7, we go to the **8th row** of Pascal's Triangle, and find the **5th number**. This will be our starting coefficient. We will expand our bracket initially like this:

${}^{7}C_{5}(3x)^3(-2)^4$

$(35)(3x)^3(-2)^4$

A common mistake for students to make here is to write:

$35 \times 3x^3 \times -2^4$

– do not fall into this trap. Be clear and set your working out clearly with use of brackets.

Continuing with the correct answer we can simplify:

$(35) \times (27x^3) \times (16)$

$15120x^3$ – the final answer.