This revision guide uses information about quadratics, trigonometry, and exponentials. These are all covered in further details in other revision areas as well.

## Domains and Range

Functions are mappings. Values go into the function and values come out of the function. The entire set of numbers that can be entered are called the *domain* and the entire set of numbers that output from the function is called the *range*.

Often in a graphical question it may be easier to assume the domain as the *x* numbers and the range as the *y* numbers.

*Domain and range from an algebra function*

f: $\rightarrow$ $2x^2 - 3$, $x \in$real number

The domain of this function is defined as $x \in$real number, which means that *x* can be any integer (whole number) value, including negative numbers. By entering different types numbers it soon becomes apparent that the smallest number that is output by the function is -3. Some values have been shown below in the table.

| Input | Output (mapping) |
|---|---|
| -5 → | 47 |
| 5 → | 47 |
| 0 → | -3 |
| 20 → | 797 |
| -30 → | 1797 |

So the domain is any value, and the range is f ≥ -3

## Inverse functions

An inverse function is one where the function's range is sent back to its domain.

For example,

$f(x) \rightarrow x+3$ and the inverse is $f(x)^{-1} \rightarrow x-3$.

Putting $x=2$ into the function will give us 5. Putting 4 into the inverse function will get us back to 2, the number we started with.

*One other point about the inverse to remember is that a function will not have an inverse if it is not a one-to-one function.*

To find an inverse function:

$$f(x) \to \frac{3+x}{4}$$

$$y = \frac{3+x}{4}$$
$$4y = 3 + x$$
$$4y - 3 = x$$
$$f(x)^{-1} = 4x - 3$$

**Composite functions**

Composite functions are when we put two or more functions together. They are often written as follows:

$$f \circ g(x)$$

This translates as: take function *g(x)* and put it into *f(x)*.

Example:

$f(x) \to x^2 + 1$ and $g(x) \to \frac{x}{4}$.

Find (a) $f \circ g$ (b) $g \circ f$

(a) $\left(\frac{x}{4}\right)^2 + 1$ (b) $\frac{x^2+1}{4}$

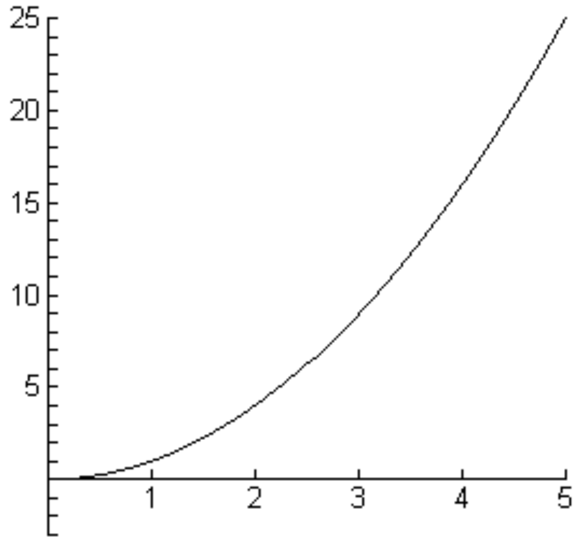
**Transformation of functions**

Functions can be transformed in the following ways: reflections; translations; and stretches.

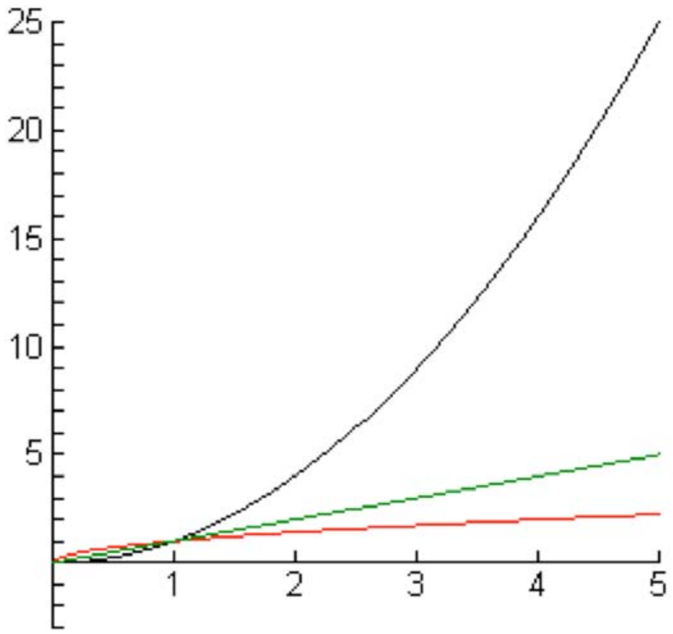
To demonstrate each transformation we will take the same function:

$f(x) \to x^2$. The function has the domain $x \geq 0$.

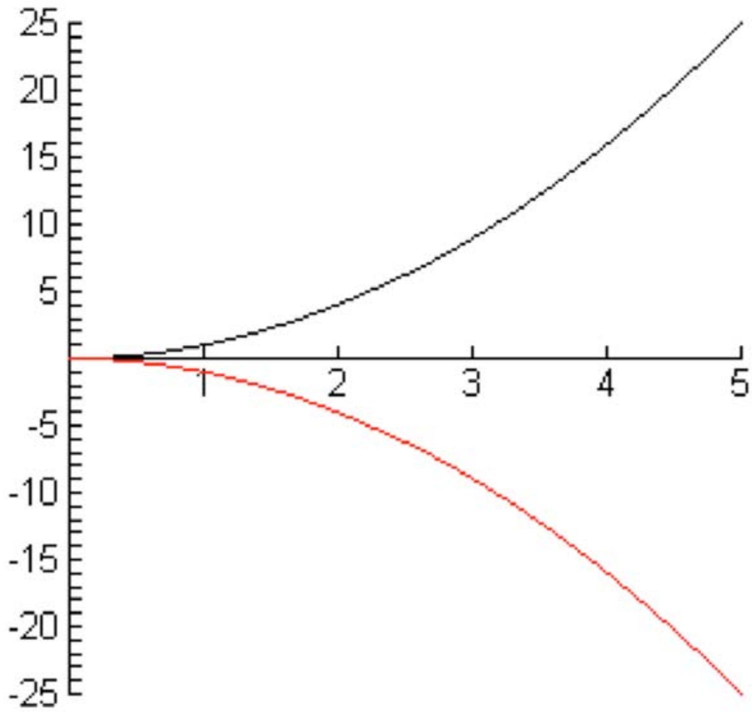
A graph of this function is as follows:



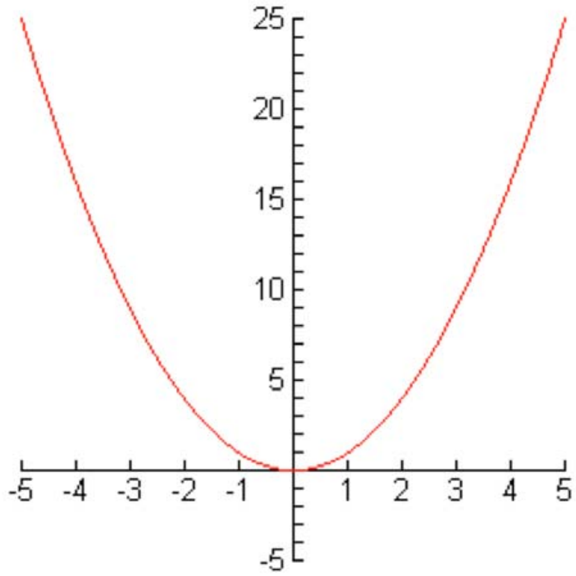

1. The inverse of the function produces a reflection in the line $y = x$.



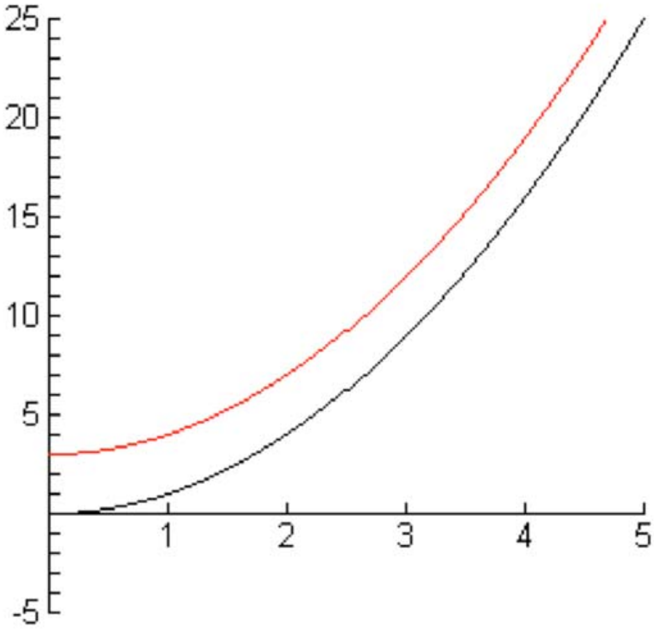

2. $-f(x)$ gives a reflection in the line $y = 0$.



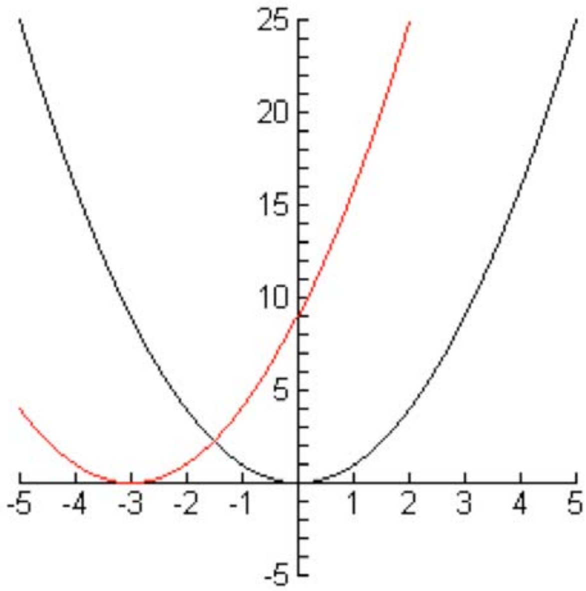

3. $f(-x)$ gives a reflection in the line $x = 0$




4. $f(x)+3$ gives a translation by the vector $\begin{pmatrix}0\\3\end{pmatrix}$.

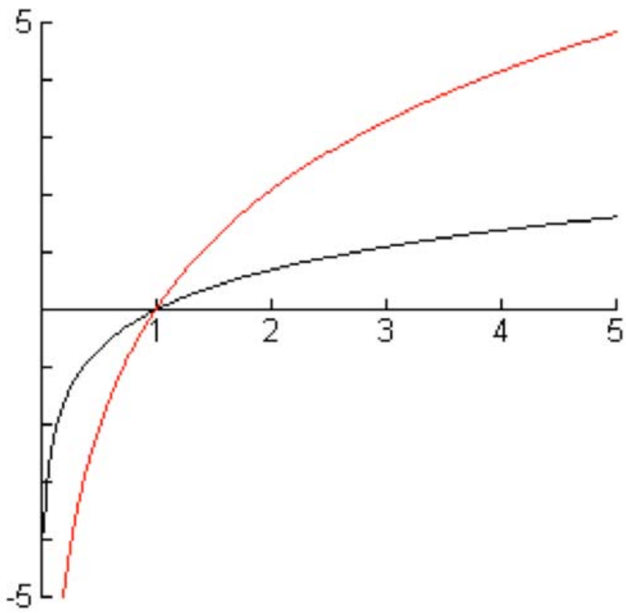



5. $f(x+3)$ gives a translation by the vector $\begin{pmatrix}-3\\0\end{pmatrix}$.

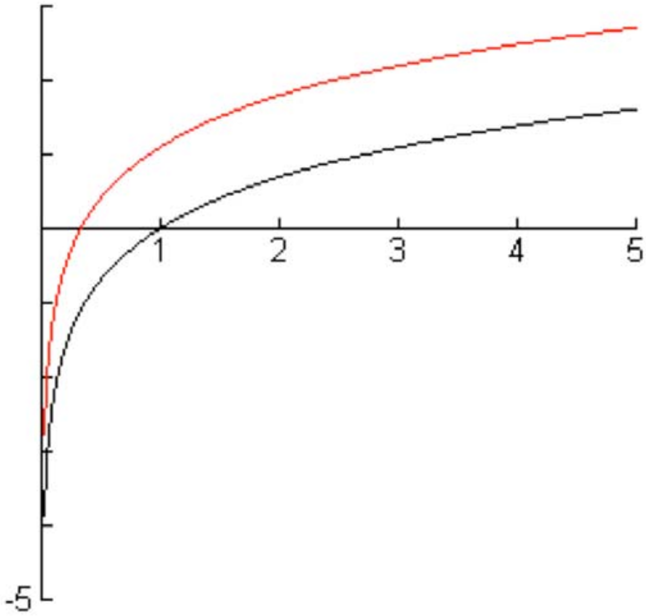



In questions 6 and 7 I have used the function $f(x) \to \ln(x)$ to illustrate the stretches.

6. $3f(x)$ gives a vertical stretch by a factor of 3.



7. $f(3x)$ gives a horizontal stretch by a factor of $\frac{1}{3}$.




**Sine and Cosine functions**

Sine and cosine curves are often used to model real life situations, such as hours of sun or tidal times. The basic sine and cosine curves are shown below.

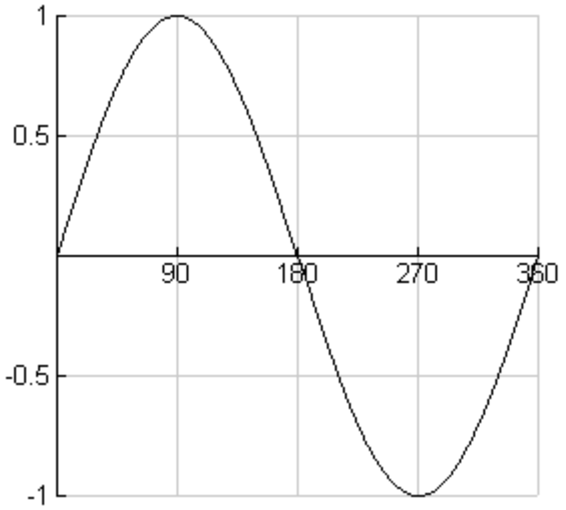
It is important to have a basic grasp of the nature of these curve and trigonometry, especially angles between 0º and 360º. Also you should be aware of the value of sin and cos that give 0 and 1. These are shown below in the table.

| | |
|---|---|
| sin 0 | 0 |
| sin 90 | 1 |
| sin 180 | 0 |
| sin 360 | 0 |
| cos 0 | 1 |
| cos 90 | 0 |
| cos 270 | 0 |
| cos 360 | 1 |

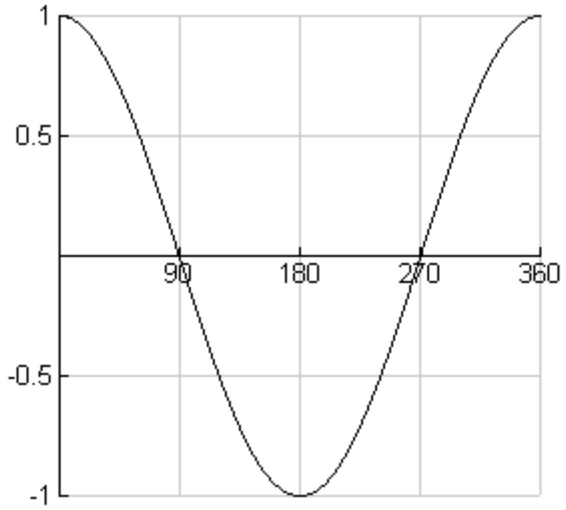
These values can be seen clearly from the sin and cos curves below.

*Basic curves*

*y* = sin *x*

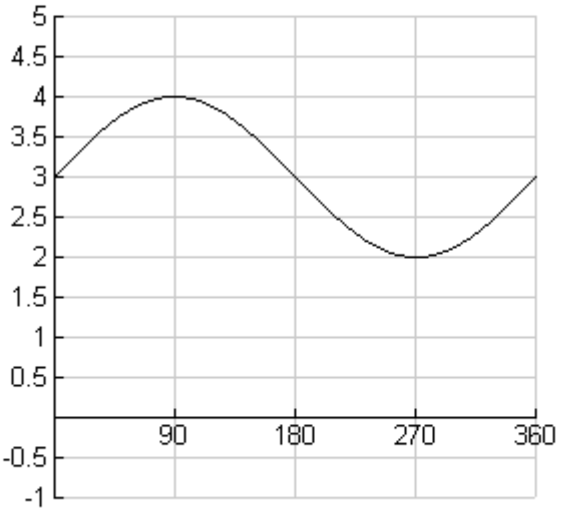



*y* = cos *x*

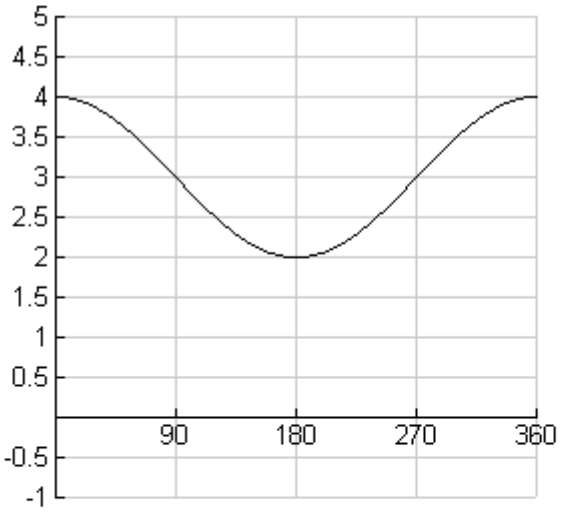



*Adding a number to the curve*

*y* = (sin *x*) + 3



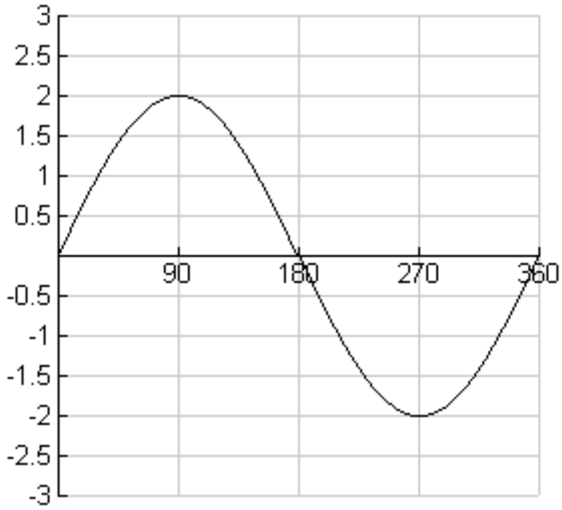

*y* = (cos *x*) + 3



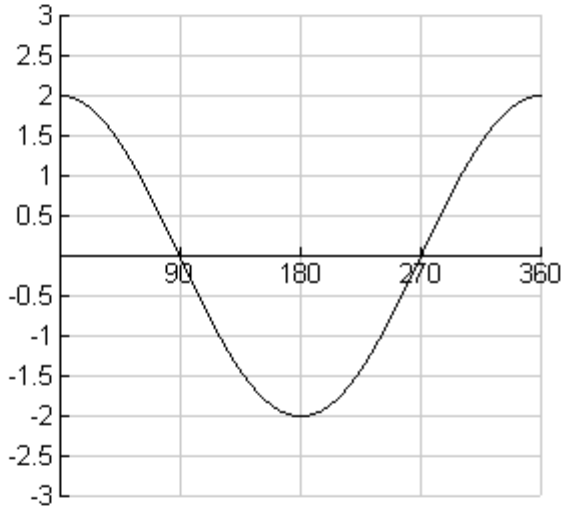

Adding a number to the curve causes the curve to translate (slide) up. Subtracting a number from the curve causes the curve to translate (slide) down.

*Multiplying by a number*

$y = 2\sin x$



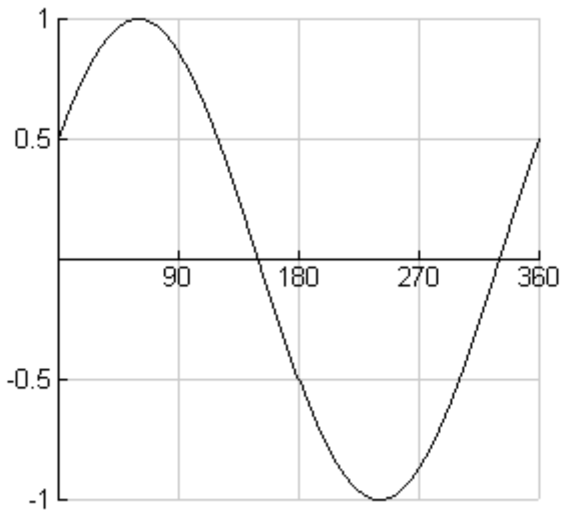

$y = 2\cos x$



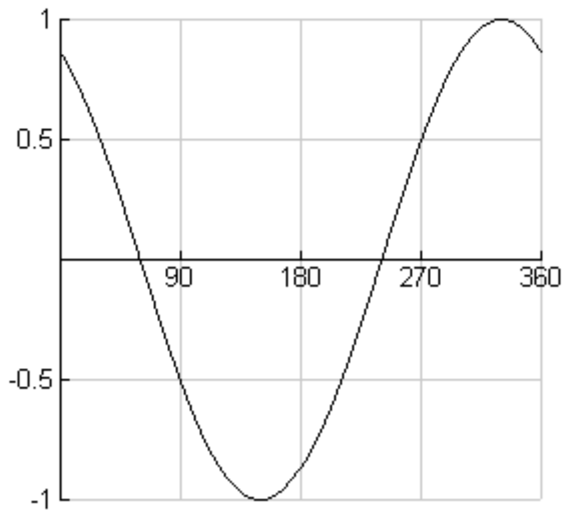

Multiplying the curve by a number causes the curve to be stretched

Adding a number to $x$

$y = \sin(x+30)$



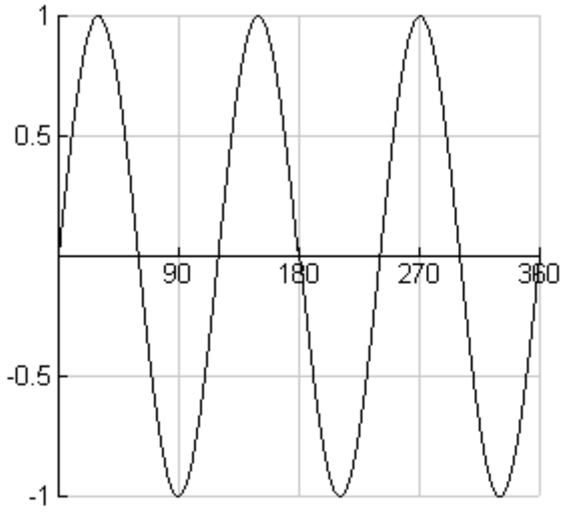

$y = \cos(x+30)$



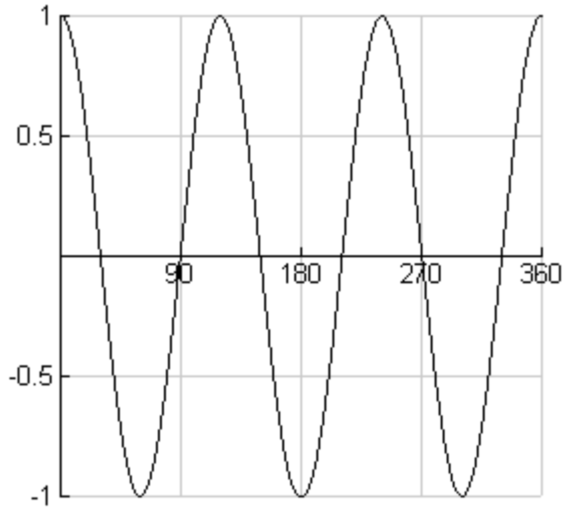

Adding a number to $x$ has caused the curve to translate to the right.

*Multiplying the x number*

$y = \sin(3x)$




$y = \cos(3x)$



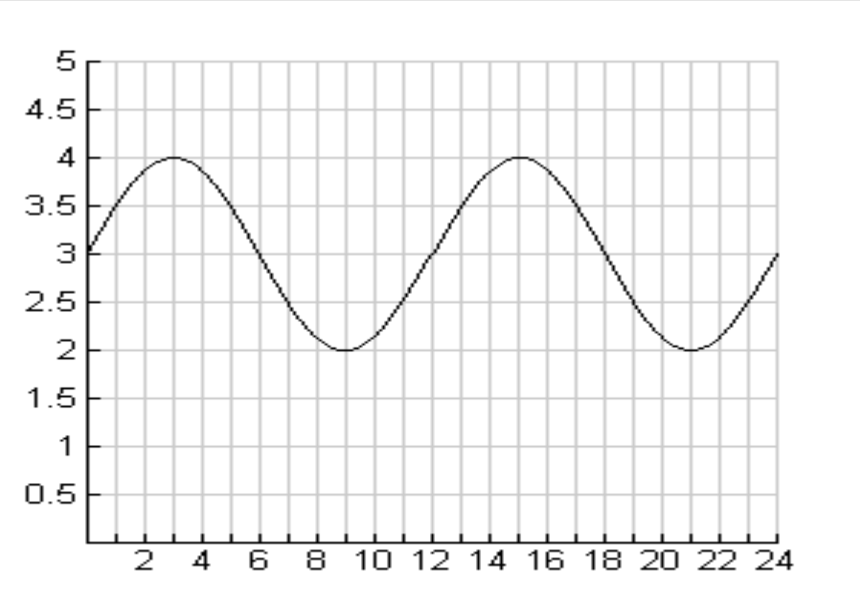

Multiplying the *x* number causes the curve to oscillate more.

More information on trig curves can be seen from the trig revision sheet. This covers the **amplitude** and **period** in more depth.

All of the above can be put together in modelling questions. Particular use should be made of the values of sin and cos that give 0 and 1. The worked example below shows all this information clearly.

**Guided example**

The height of the water in the harbour of Trumpton is tidal and modelled on a sine curve as shown in the diagram below.




The curve has the equation:

$$h = \sin(pt) + q$$

where $t$ = hours after midnight and $h$ = height of the water in metres.

(a) Use your graph to find the height of the water at 10 AM.

(b) A yacht can only safely enter and leave the harbour when there is more than 3 metres of water in the harbour.
Give the times of day when the ship can safely enter and leave the harbour.

(c) Find the values of $p$ and $q$.

Answer (a)

Answer (a) and (b) can be found by simply reading the graph. At 10 AM $t$ = 10, so there is approximately 2.2 metres of water in the harbour.

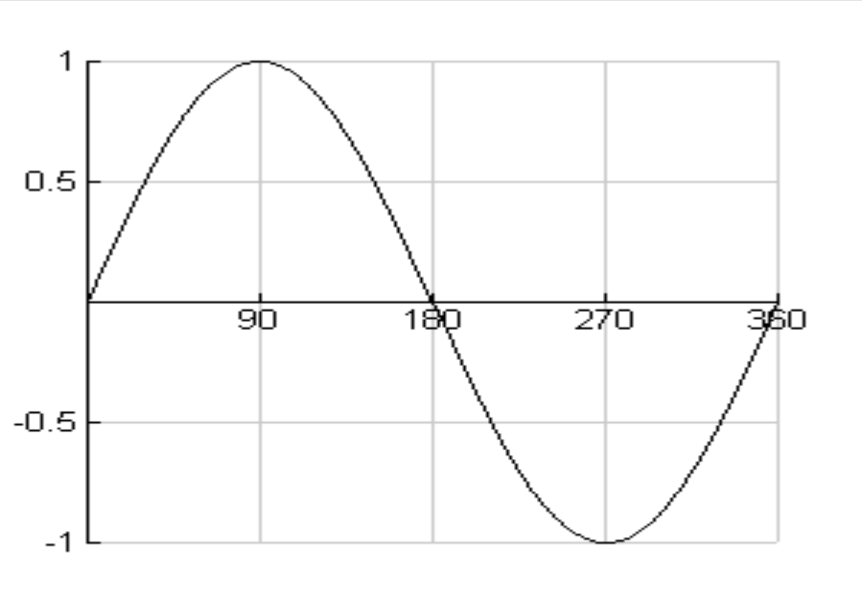
Answer (b)

By looking at the graph the height is above 3 for $0 \le t \le 6$ and $12 \le t \le 18$.
The corresponding times are:
Midnight to 6 AM and 12 PM and 6 PM.

Answer (c)
Since the curve has the equation $h = \sin(pt) + q$ we want to make use of $\sin 0^\circ = 0$. Looking at making $\sin(pt) = \sin(0)$, we can make $t = 0$. It does not matter what the value of $p$ is by taking $t = 0$ the $\sin(pt)$ becomes 0.
When $t = 0$, $h = 3$, so we have the equation:

$$3 = \sin(0) + q$$
$$q = 3$$

Using this value and making use of sin 90 = 1. Remember the sine curve features. It may be useful to draw the $y = \sin x$ on your TI and compare it to the curve in the question.




When sin 90 = 1 this is the top of the graph. In the modelled graph the curve is at it's peak when $t = 3$.

We only need take the $\sin(pt) = 1$, so $pt = 90^\circ$ and we know that $t = 3$, so $p$ is 30.

By drawing the curve of $y = \sin(30x) + 3$ on your TI you can check to see if it is the same as the curve given in the question (it is!).

$\Rightarrow$ $p = 30$ and $q = 3$

## Exponential functions

Remember:

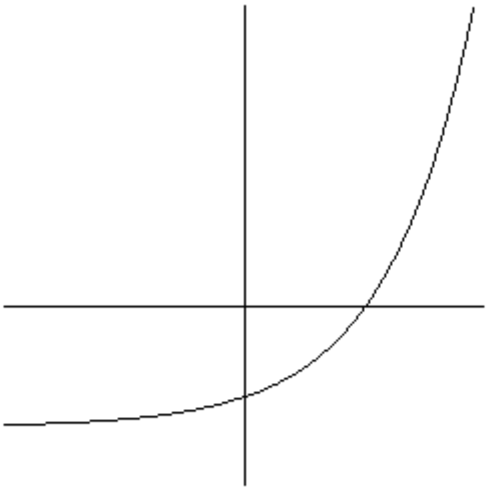
$x^0 = 1$

$x^1 = x$

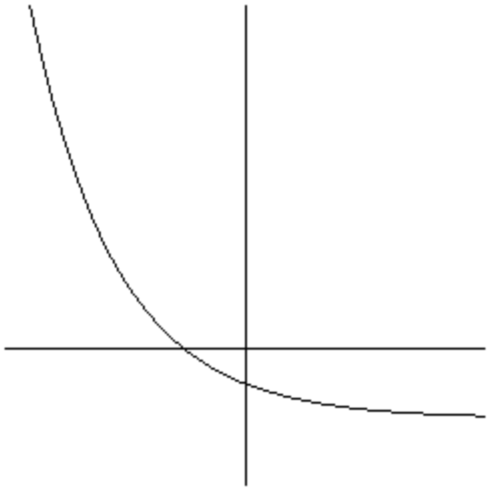
Exponential functions are functions where the *x* number becomes a power. The functions below are all examples of exponentials.

$y = 2^x$ $y = 3^{-z}$ $y = \frac{1}{2}^x + 3$

An exponential curve looks like:

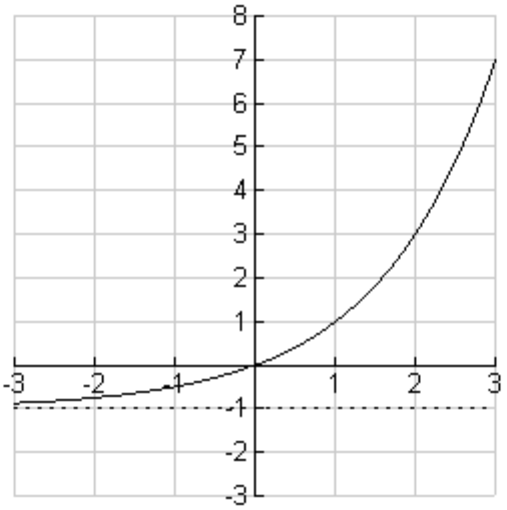


A negative exponential curve is:



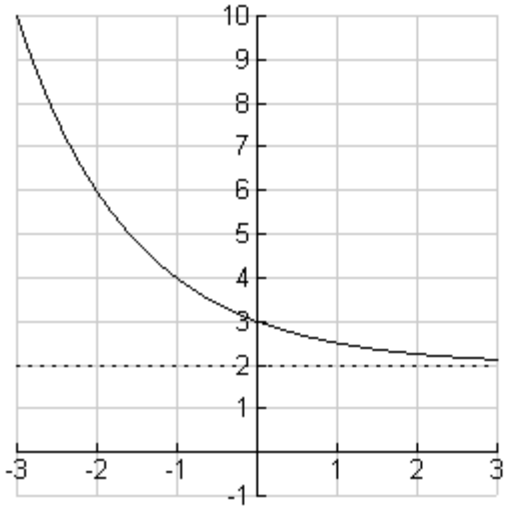
Exponential curves have *asymptotes*. These are simply lines that the curve is heading towards but will never actually reach. In the two diagrams below the asymptotes have been drawn on in dotted lines with the equation given below.

Example 1: $y = 2^x - 1$




Asymptote: $y = -1$

Example 2: $y = 2^{-x} + 2$

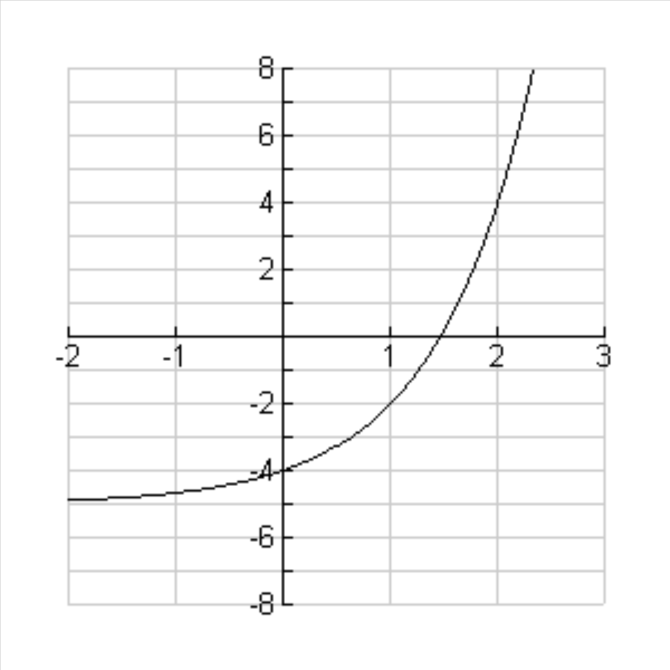



Asymptote: $y = 2$

Look at the table opposite which shows a number of exponential functions and their asymptotes. Look for the pattern and you can get the next two asymptotes.

| Function | Asymptote |
|---|---|
| $y = 2^x + 3$ | $y = 3$ |
| $y = 3^{-x} - 5$ | $y = -5$ |
| $y = e^x - 2.5$ | $y = -2.5$ |
| $y = 0.5^x$ | $y = 0$ |
| $y = e^x + 0.5$ | $y = 0.5$ |
| $y = 2^x + 3$ | $y = ?$ |
| $y = 2^{-x} - 8$ | $y = ?$ |

**Guided example**
The diagram below shows the function $y = b^x + a$.




The curve passes through the coordinates (0, -4) and (1, -2).
(a) Find the value of $a$.
(b) Find the value of $b$.
(c) Find the equation of the asymptote of the curve.

Answer (a)
Using the first coordinate we can substitute in values of $x$ and $y$ into the equation as follows:
$$-4 = b^0 + a$$
Since $b^0 = 1$ we now have:
$$-4 = 1 + a$$
$$a = -5$$

Answer (b)
Now using the second set of coordinates:
$$-2 = b^1 - 5$$
Since $b^1 = b$ we now have:
$$b = 3$$

Answer (c)
The asymptote can be found by ignoring the power of $x$ part of the function, so the asymptote will be:
$$y = -5$$