Probability formula

The following formula is taken from the IB formula booklet:

Probability of an event A:	$\mathrm{P}(A)=\dfrac{n(A)}{n(U)}$
Complementary events:	$\mathrm{P}(A')=1-\mathrm{P}(A)$
Combined events:	$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$
Mutually exclusive events:	$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)$
Independent events:	$\mathrm{P}(A\cap B)=\mathrm{P}(A)\,\mathrm{P}(B)$
Conditional probability:	$\mathrm{P}(A\mid B)=\dfrac{\mathrm{P}(A\cap B)}{\mathrm{P}(B)}$

All the formulae are explained below and can be seen in the guided example below the explanations.

Probability of an event
The probability of event must be written as a number between 0 and 1, and written as a fraction, percentage or decimal. It is easiest to use a fraction if possible.
The first formula simply means put the number of times A occurs as the numerator (top of the fraction) and the number of all the events as the denominator (the bottom of the fraction).

Complentary events
This is simply explaining that if the probability of A is $\frac{1}{3}$ then the probability of A' (not A happening is 1 - $\frac{1}{3}$ = $\frac{2}{3}$.

Combined events
This formula can be best understood by using the set diagrams. In words it simply means add up the number of times A occurs with the number of times B occurs and take away the number of times both occur together.
For example: If 80 people choose Biology, 60 choose Chemistry and 20 choose both Biology and Chemistry, then $\mathrm{P}(A\cup B)$ = 80 + 60 – 20 = 120.

Conditional probability
This is used when we see the 'given that' in the question. P(A/B) means the probability of *A* given *B*.
When we are 'given' something in probability questions we can cut down the outcomes we have to only things where *B* has occurred. This forms the bottom of the fraction. The top of the fraction is where *A* and *B* have occurred.
This is shown in the guided example below.

Outcomes or combinations
These are simply all the possible things that can happen. For example when a die is rolled there are 6 outcomes. This will form the bottom of the fraction in an answer to a probability question.

Guided example
A unbiased 6-sidced dice is marked with the numbers: 2, 2, 3, 4, 5, and 8.

The die is rolled. Find the probability of the die landing on:
(a) landing on a prime,
(b) not landing on a 4,
(c) landing on a 2 given that it has landed on a prime.

Answer (a)
This uses the formula $P(A) = \frac{n(A)}{n(U)}$.
n(A) = 4 (prime numbers 2, 2, 3, 5)
n(U) = 6 (six outcomes on a dice)

$\Rightarrow$ the answer is $\frac{4}{6}$.

Answer (b)
This uses the formula $P(A') = 1 - P(A)$.
1 - $\frac{4}{6} = \frac{2}{6}$.

Answer (c)
The important part of this question is the 'given that' which indicates that the bottom of the
fraction is now only 4, the prime numbers being 2, 2, 3, and 5. As there are two 2's, this will go onto the top of the fraction giving the answer $\frac{2}{4}$.

Probability from tables

Frequently probability questions are often given in tables. These questions are reasonably easy to answer. The conditional probability question is the only one that sometimes catches people out.

The best way to see these is by using the guided example below.

Guided example

The table below shows the choices made by males and females for their IB second language.

	Female	Male	Total
French	60	20	80
Spanish	50	70	120
	110	90	200

A student is chosen at random. Find the probability that the student is:

(a) female,
(b) male and studies French,
(c) a female, given the student studies Spanish.

Answer (a)

The total is 200 so that will become the denominator of the fraction. There are 110 females so we have the probability:

$$\frac{110}{200} = \frac{11}{20}$$

Answer (b)

There are 20 students who are male and who study French so we have the probability:

$$\frac{20}{200} = \frac{1}{10}$$

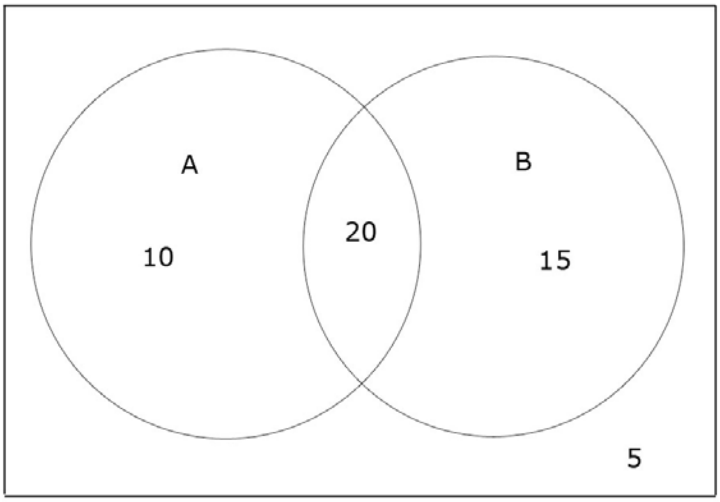
Answer (c)

The 'given' turns this into a conditional probability question. As we now know the student studies Spanish we look for the total number of students who study Spanish and see it is 120, this becomes the denominator. Finding how many of these 120 are female students who study Spanish we can see it is 50. So the probability is:

$$\frac{50}{120} = \frac{5}{12}$$

Venn diagrams and sets

Much of the probability formula is based on set notation. Looking at the Venn diagram below we can see the numbers that represent the various parts of the formulae used above.



$nP(U) = 50$ $nP(A) = 30$ $nP(B) = 35$ $n(A \cap B) = 20$ $nP(A \cup B) = 45$

Using the above we can give the probabilities:

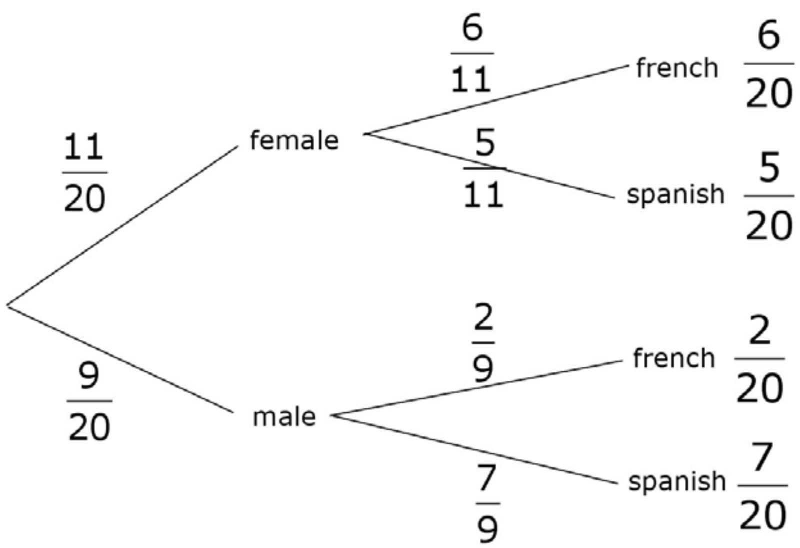
1. $P(A) = \frac{30}{50} = \frac{3}{5}$
2. $P(A') = 1 - \frac{3}{5} = \frac{2}{5}$
3. $P(A \cup B) = \frac{30}{50} + \frac{35}{50} - \frac{20}{50} = \frac{45}{50} = \frac{9}{10}$
4. $P(A/B) = \frac{20}{35} = \frac{4}{7}$
5. $P(B/A) = \frac{20}{30} = \frac{2}{3}$

Tree diagrams

Tree diagrams are occasionally used by the IB in examinations. Tree diagrams are a clear way to view all the outcomes, and their associated probabilities when 2 or more events occur.

You may be asked to draw a tree diagram or to copy and complete a tree diagram. From this diagram you can be asked for simple probabilities or even a conditional probability.

The information in the guided example above about students' IB language choices has been used to produce a tree diagram.



Points to remember:

- Each set of branches' probabilities must add up to 1. These probabilities should be filled in on the line, not at the end or the beginning of the line.
- It is useful to put the combined probabilities at the end of the tree diagram. This will help when answering any probability questions that will arise.
- Each combined outcome is gained by multiplying the probabilities on the lines.

  So the first outcome which is female studying French is $\frac{11}{20} \times \frac{6}{11} = \frac{6}{20}$
- All the fractions (or decimals) at the end of the tree diagram add up to 1.
- If you are using fractions in the tree diagram use the same denominator at the end of diagram for each outcome. It will help when answering tree diagram questions.

Now we can answer probability questions associated with the tree diagram.

A student is chosen at random. Find the probability that the student is:

(a) a female,
(b) a student who studies French,
(c) a male who studies French,
(d) a female, given that the student studies Spanish.

(a) a female is simply the $\frac{11}{20}$ from the female line.

(b) a student who studies French can be gained by adding together the two lines that have French in them: $\frac{6}{20}+\frac{2}{20}=\frac{8}{20}$.

(c) a male who studies French can be found by looking at the only outcome with male and French: $\frac{2}{20}$.

(d) a female, given that the student studies Spanish:
To answer the 'given that', conditional probability question from a tree diagram we need to look only at the Spanish lines to find the denominator of the fraction: $\frac{5}{20}+\frac{7}{20}=\frac{12}{20}$.

Now to find the numerator we look at Female and Spanish which is $\frac{5}{12}$.

So the final probability will be:

$$\frac{(\frac{5}{20})}{(\frac{12}{20})}=\frac{5}{12}.$$

Note: by keeping the same denominator at the end of the tree diagram these conditional probabilities become easier to answer.

Binomial Distribution

The binomial distribution can be used when there are a known number of trials and only 2 outcomes to each trial.

It is written as X~B(*n, p*), where *n* is the number of trials and *p* is the probability of a success.

The expected (mean) outcome of a binomial distribution is *np* and then standard deviation of a binomial distribution is $\sqrt{np(1-p)}$. You may see (1-*p*) referred to as *q*.

When calculating probabilities in a binomial distribution you must remember how many combinations of an event happening there are. These can be calculated using the ${}^{n}C_{r}$ function of your GDC or by referring to Pascal's triangle (see the binomial expansions sheet).

This is best explained in the guided example below.

Guided example

Eddie the eagle-eyed archer can hit the bulls eye with a probability of $\frac{1}{3}$. Each arrow he fires is independent of previous arrows.

Eddie fires 5 arrows in a practice session and records the amount of bulls eyes he hits.

(a) Find the mean and standard deviation of the number of bulls eyes Eddie hits.

(b) Find the probability of Eddie getting:
(i) no bulls eye,
(ii) a bulls eye only with his 3rd throw,
(iii) one bulls eye.

(a) $X \sim B(5, \frac{1}{3})$

Mean $= 5 \times \frac{1}{3} = \frac{5}{3}$

Standard deviation $= \sqrt{5 \times \frac{1}{3} \times \frac{2}{3}} = \sqrt{\frac{10}{9}}$

(b)

(i) no bulls eyes is a simple probability of $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^5 = 0.132$

(ii) only with his third throw will be $\frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right) = 0.066$

(iii) one bulls eye we have to use the ${}^nC_r$ function as follows:

$${}^5C_3 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^3 = 10\left(\frac{4}{243}\right) = 0.165$$