Calculus test IB HL

1. A particle is moving in a straight line with velocity given by,

   $v(t) = -t^2 + \frac{3t}{2} + 5$, where t is time in seconds and v is metres per second.

   a) Find the particle's maximum velocity, and confirm by using calculus that this value is a maximum. *[4 marks]*

   b) Find the acceleration at 5 seconds.

   What does your result tell you about the motion of the particle when $t=5$? *[3 marks]*

   c) After 1 second the particle has been displaced by $\frac{65}{12}$ metres.

   Find an expression for the displacement, S, in metres in terms of time, t. *[4 marks]*

2. Find the exact x-coordinate of the point of inflection of $y = x^3e^x$ where $x > -5$. *[6 marks]*

3. A radioactive substance, R, is given by the equation,

   $\frac{dR}{dt} = -kR$.

   Initially there are 100 grams of the substance present.

   a) Solve $\frac{dR}{dt} = -kR$ to find R in terms of t and k. *[4 marks]*

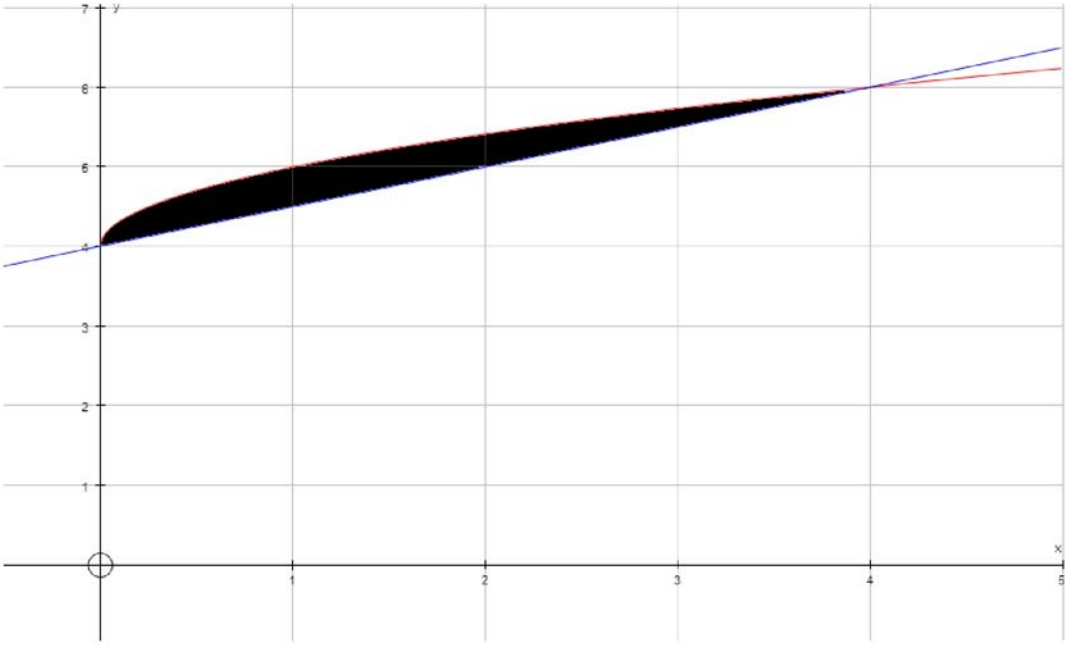
   b) The value of R halves every 50 years. Show that the value of k can be written as,

   $$k = \frac{\ln 2}{50}$$ *[3 marks]*

   c) Find the amount of grams of R present after 225 years. *[2 marks]*

4. The diagram below show the two functions,

$f(x) = 4 + \sqrt{x}$ and $g(x) = \frac{1}{2}x + 4$, where $0 \leq x \leq 5$.



a) Find the area created between the two functions, shown in the diagram as the shaded region. *[4 marks]*

b) $f(x)$ is rotated through 2π radians about the x-axis.

Find the volume created by the function between $x=2$ and $x=4$. Give your answer in terms of π. *[4 marks]*

5. Integrate $\int \frac{dx}{\sqrt{25 - x^2}}$ by using the substitution $x = 5\sin u$.

Show all your working. *[5 marks]*

6. Find the equation of the tangent to the curve $2x^3 - 3y^2 + x = 0$ at the point (1,1). Give your answer in the form $ax + by + c = 0$.

[6 marks]

7. Show that $\int \left(e^x \sin x\right)dx = \frac{e^x(\sin x - \cos x)}{2} + c$. *[5 marks]*

Answers

1. a) $\frac{89}{16}$ m/s^{-1}.

   Ensure that $\frac{d^2V}{dt}$ has a negative value.

   b) $-\frac{17}{2}$ m/s^{-2}.

   The particle is decelerating.

   c) $S = -\frac{t^3}{3} + \frac{3t^2}{4} + 5t$

2. 0

3. a) $R = 100e^{-kt}$

   c) 4.42 grams

4. a) $1\frac{1}{3}$ units^2

   b) 65.9π units^3

5. $\arcsin\left(\frac{x}{5}\right) + c$

6. $7x - 6y - 1 = 0$