Calculus test IB SL

1. A particle is moving in a straight line with velocity given by,

   $v(t) = -t^2 + \frac{3t}{2} + 5$, where t is time in seconds and v is metres per second.

   a) Find the particle's maximum velocity, and confirm by using calculus that this value is a maximum. *[4 marks]*

   b) Find the acceleration at 5 seconds.

   What does your result tell you about the motion of the particle when $t=5$? *[3 marks]*

   c) After 1 second the particle has been displaced by $\frac{65}{12}$ metres.

   Find an expression for the displacement, S, in metres in terms of time, t. *[4 marks]*

2. Find the exact x-coordinate of the point of inflection of $y = x^3e^x$ where $x > -5$. *[6 marks]*

3. The function f is given by $f(x) = 3\sin(6x - 5)$.

   a) Find $f''(x)$. *[4 marks]*

   b) Write down $\int f(x)dx$. *[2 marks]*

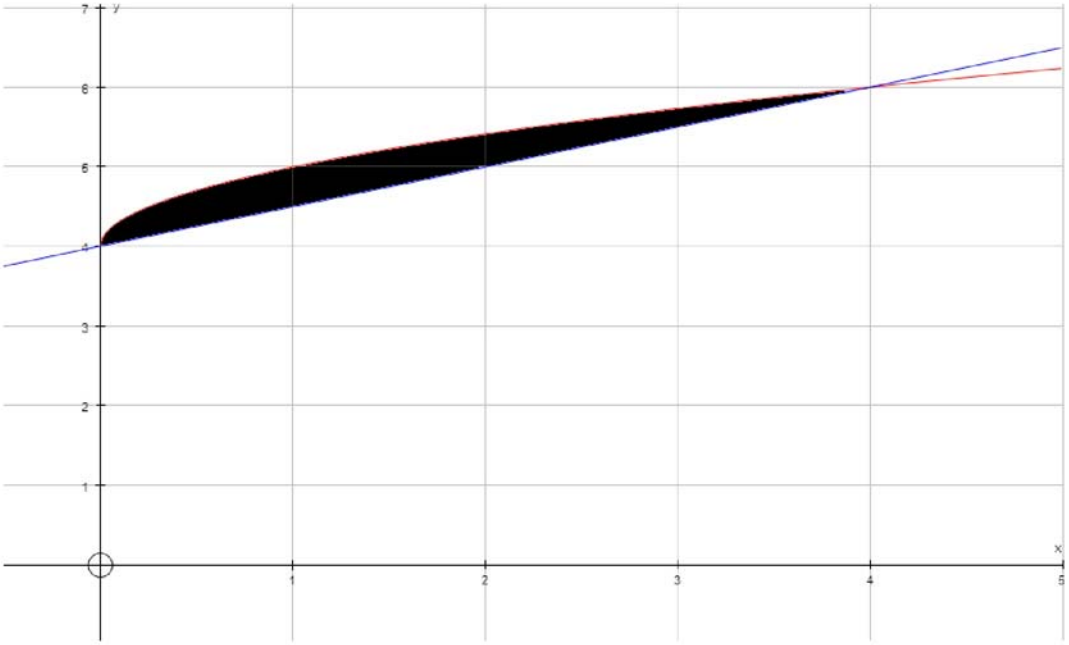
4. An equation of a curve is given as $y = x^3 - 2x^2 + 3x + 1$.

   a) Find the gradient function, $\frac{dy}{dx}$, and hence find the gradient of the curve at the point where $x=2$. *[3 marks]*

   b) Find the equation of the normal to the curve at the point where $x=2$, giving your answer in the form $ax + by + c = 0$. *[4 marks]*

5. The diagram below show the two functions,

$f(x) = 4 + \sqrt{x}$ and $g(x) = \frac{1}{2}x + 4$, where $0 \leq x \leq 5$.



a) Find the area created between the two functions, shown in the diagram as the shaded region. *[4 marks]*

b) $f(x)$ is rotated through 2π radians about the x-axis.

Find the volume created by the function between $x=2$ and $x=4$. Give your answer in terms of π. *[4 marks]*

6. Find the solution to $\int_1^2 (2x-3)^4 \, dx$. *[4 marks]*

Answers

1. a) $\frac{89}{16}$ m/s^{-1}.

   Ensure that $\frac{d^2V}{dt}$ has a negative value.

   b) $-\frac{17}{2}$ m/s^{-2}.

   The particle is decelerating.

   c) $S = -\frac{t^3}{3} + \frac{3t^2}{4} + 5t$

2. 0

3. a) $-108\sin(6x - 5)$

   b) $-\frac{1}{2}\cos(6x - 5) + c$

4. a) $\frac{dy}{dx} = 3x^2 - 4x + 3$ and gradient = 7

   b) $x + 7y - 51 = 0$.

5. a) $1\frac{1}{3}$ units^2

   b) 65.9π units^3

6. $\frac{1}{5}$ units^2