

Tangents and points of inflexion

IB SL/HL

1. The function $f(x)$ is defined as $f(x) = \ln(x^2 + 5)$.
 - a) Find the exact minimum coordinates of the graph.
 - b) Find the equation of the tangent when $x = 1$.
 - c) Find the two coordinates of the two points of inflexion of the curve.

2. The function $f(x)$ is defined as $f(x) = x^4 - x^2 + 2x$.
 - a) Find the minimum coordinate of the function.
 - b) Show that the two points of inflexion are $P_1\left(\frac{1}{6}, \frac{469}{1296}\right)$ and $P_2\left(-\frac{1}{6}, -\frac{395}{1296}\right)$.
 - c) Find the equation of the straight line that passes through the two points of inflexion.

3. $f(x) = e^{(2x-3)} + 5$.
 - a) Show by use of algebra that there is no minimum point of this function.
 - b) Find the gradient of the curve at the point where $x = 4$.

4. The function $f(x)$ is defined as $f(x) = x \ln(5x^2 + 1)$.
 - a) Find $f'(x)$.
 - b) Hence show that there are no minimum points of the function.
 - c) Show by use of algebra that the point of inflexion of the function is $(0,0)$.

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5. The function $f(x)$ is defined as $f(x) = x + \ln(3x^2 + 1)$.
- Find the equation of the tangent to the function at the point where $x = 1$.
 - Find the minimum coordinates of the function.
 - Find the exact coordinates of the points of inflexion of the function.
6. The function $f(x)$ is defined as $f(x) = \frac{x+1}{e^x}$.
- Find the maximum point of the function.
 - Find the exact coordinate of the points of inflexion.
 - Find the equation of the straight line that connects the maximum point to the point of inflexion.

Answers

1. a) $(0, \ln(5))$
- b) $y = \frac{x}{3} + 3\ln 6$
- c) $(\sqrt{5}, \ln(10))$ and $(-\sqrt{5}, \ln(10))$
2. a) $(-1, -2)$
- c) $y = 2x + \frac{7}{1296}$
3. a) $f'(x) = 2e^{(2x-3)}$ and $f'(x) \neq 0$.
- b) $2e^1$

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4. a) $f'(x) = \frac{10x^2}{5x^2 + 1} + \ln(5x^2 + 1).$

b) Hence, $10x^2 + (5x^2 + 1)\ln(5x^2 + 1) = 0$ is not possible as both parts of the equation will give positive answers.

c) $f''(x) = \frac{20x(5x^2 + 1) - 100x^3}{(5x^2 + 1)^2} + \frac{10x}{(5x^2 + 1)}$

make $f''(x) = 0$ and this simplifies to: $5x^3 + 3x = 0.$

Only solution is (0,0).

5. a) $y = \frac{5}{2}x + \ln(4) - \frac{3}{2}$

b) (-0.184, -0.088)

c) $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} + \ln(2)\right)$ and $\left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} + \ln(2)\right)$

6. a) (0,1)

b) $\left(1, \frac{2}{e^1}\right)$

c) $y = \left(\frac{2}{e^1} - 1\right)x + 1$