

1. Explain clearly what is meant by,
 - a) a type I error,
 - b) a type II error.

2. A random variable is normally distributed with a mean of 120 and a variance of 64. New data has been found about the variable and it is thought that the mean is now greater than 120.
 - a) Write suitable hypotheses.
 - b) A sample of size 50 is taken and it is decided to reject the null if sample mean is greater than 122. Find the probability of a type I error being made under these conditions.
 - c) It is later found that the mean is in fact 121.5, but the variance does not change. When a sample of size 50 is taken, calculate the probability of a type II error.

3. On a recent cricket tour to England the Australian captain, Ricky Punter, lost 11 out of 12 tosses of the coin at the start of each match. He called 'tails' for each toss. The Australians decided to complain that the coin was biased towards heads and demanded it was tested.
 - a) Write suitable hypotheses for the test.
 - b) The coin is tossed 100 times and it was decided that if there 60 or more heads then the coin will be declared as biased. Calculate the probability of a type I error.
 - c) The probability of a type I error is decided to be set to 5%. Calculate the least number many heads required to reject the null hypothesis.
 - d) Later tests discover the probability of a head is in fact 0.65. Calculate the probability of making a type II error when doing the test in b).

Type I and II errors

IB HL Options Stats

4. An eight-sided die is thought to be biased towards the eight and two hypotheses are put forward:

$$H_0 : \mu = \frac{1}{8} \quad \text{and} \quad H_1 : \mu > \frac{1}{8}$$

The die is rolled 80 times and if 16 or more 8's are recorded, the null hypothesis will be rejected.

- a) Find the probability that (H_1 is accepted/ H_0 is true).
- b) Later the die is in fact found to be biased such that the probability of the eight-sided die landing on an eight is in fact $\frac{1}{4}$.

Find the probability that (H_0 is accepted/ H_1 is true).

5. The amount of brain cells burnt off daily by IB students has been analysed in the past and it is known to follow a normal distribution with a mean of 140 and a variance of 2500. Two hypotheses are put forward:

$$H_0 : \mu = 140, H_1 : \mu > 140$$

In a certain school, the students are thought to be overworked and the amount of brain cells being burnt off is thought to be more than 140. A sample of 50 students in the IB years is taken and it is decided that if more than 165 brain cells are burnt off daily than the null hypothesis will be rejected.

- a) Calculate the probability of a type I error.
- b) Scientists later analyse the entire IB year and find that the number of brain cells being burnt off is in fact 172. Calculate the probability of a type II error.

Answers

1. a) A type I error is when the null is rejected when in fact it is true.
b) A type II error is when the null is accepted when in fact it is false.
2. a) $H_0 : \mu = 120$ and $H_1 : \mu > 120$
b) $p=0.039$
c) $p=0.67$
3. a) $H_0 : \mu = 50$ and $H_1 : \mu > 50$
b) $p=0.029$
c) 59 or more
4. a) $p=0.0315$
b) $p=0.1226$
5. a) $p=0.0169$
b) $p=0.161$