

1. The continuous random variable X is uniformly distributed over the interval $x=-4$ to $x=12$.
 - a) Find the mean and variance of the distribution.
 - b) Find the probability that $x < 8$.
2. In an insurance company the telephone rings randomly at a rate of 2 calls in a 10-minute period.
 - a) Explain why the distribution can be modeled by the Poisson distribution.
 - b) Find the probability of 3 calls in a 10-minute period.
3. The probability of widget being defected is known to be 0.1. Each widget is tested.
 - a) Find the probability that the first defective widget will be the 5th one tested.
 - b) Find the mean and variance of the first defective widget.
4. A die is rolled until 3 fives are obtained.
 - a) Find a suitable model for this distribution and hence find the mean and variance of the distribution.
 - b) Find the probability that 10 rolls of the die are required.
5. In a school of 900 pupils, it has been calculated that 150 students are overweight. A sample of 30 students is chosen randomly.
 - a) Find a suitable model for this distribution and hence find the mean and variance of the distribution.
 - b) Find the probability that in this sample exactly 5 students are overweight.

6. The life of a brand of tyres is noted by the variable X , in thousands of km. It is an exponential distribution with density function,

$$f(x) = \frac{1}{3} e^{-\frac{x}{3}}, x \geq 0$$

- a) Calculate the mean and variance of the lifespan of a tyre in km.
- b) Find the probability that a tyre has a life span of more than 4000 km.
- c) Given that the tyre has a lifespan of more than 4000 km, calculate that it will need to be replaced before it has covered 5000 km.
- d) Find the median lifespan of a tyre.
7. A man has a sock draw that contains 5 black socks and 10 red socks. The man picks out socks one at time, randomly, and does not replace them. The man wishes to wear red socks,
- a) The man chooses 5 socks, calculate the probability that he has exactly 2 red socks from his 5 socks.
- b) The man chooses 6 socks, calculate the probability that he has exactly 2 red socks from his 6 socks chosen.
8. An exponential density function has a mean probability of 6 hours.
- a) Write down the variance of the density function.
- b) Write down the full continuous probability density function and hence calculate the median probability.
9. The probability of an archer hitting the Bulls eye is given as 0.3. Each attempt made is independent from previous attempts.
- a) Find the probability of the archer hitting the bulls eye for the first time with her fifth arrow.
- b) Find the probability that the archer hits the bulls eye for the third time on her twelfth arrow.

Answers

1. a) $\mu = 4, \sigma^2 = 21\frac{1}{3}$ b) $\frac{3}{4}$
2. a) Random distribution in a fixed time period.
b) 0.18
3. a) 0.0651 b) $\mu = 10, \sigma^2 = 90$
4. a) Negative binomial $\mu = 18, \sigma^2 = 90$
b) 0.0465
5. a) Hypergeometric $\mu = 5, \sigma^2 = 4.03$
b) 0.195
6. a) Mean = 3000 km Variance = 9000 km
b) 0.264
c) 0.284
d) 2079 km
7. a) 0.15 b) 0.045
8. a) 36 hours
b) $\begin{cases} \infty \\ 0 \end{cases} f(x) = \frac{1}{6} e^{-\frac{x}{6}}, x \geq 0$ Median = 4.16 hours
9. a) 0.07203
b) 0.0599