

Sampling and the Central Limit Theorem IB HL Stats Option

1. A darts player practices hitting the treble twenty. He finds that he can hit the treble twenty with a probability of $\frac{1}{5}$. He throws nine darts that he uses per 'game'.

 - a) If the player plays 50 games, find the probability that the sample mean lies between 1.7 and 1.9.
 - b) If the player plays 100 games, find the probability that the sample mean lies between 1.7 and 1.9.
 - c) Explain what happens to the probability of the sample mean being between 1.7 and 1.9 as the sample size becomes bigger.
2. On a stretch of road in France it was recorded that on average there were 1.2 accidents per week. A record was kept for each week throughout the next year.

 - a) Find the sample mean per week for the year.
 - b) Find the standard error of the sample mean for the year.
 - c) Find the probability that will be, on average, less than 1 accident per week.
3. An exponential distribution is defined as,

$$f(x) = \frac{1}{3} e^{-\frac{x}{3}}, x \geq 0$$
 - a) Write down the mean and variance of this distribution.
 - b) A random sample of 100 is taken from this distribution. Find the probability that the sample mean is less than 2.5.
4. If a large number of samples of size n are taken from a $X \sim B(10, 0.4)$, and approximately 20% of the sample means are greater than 4.1683 find the value of n .

Sampling and the Central Limit Theorem

IB HL Stats Option

5. The running per match of a midfield football player in the Premiership is known to be normally distributed with a mean of 7 km and a variance of 6.25 km. A sample of 16 random players is taken from the midfield players.
 - a) Explain why the central limit theorem is not required for this question.
 - b) Find the sample mean and the standard error of the mean of the samples.
 - c) Find the probability that the mean of these 16 players lies between 6.5 km and 7.5 km.
6. A distribution is rectangular such that $X \sim U(0,8)$. Sixty samples are taken from this sample. Find the probability that the sample mean is less than 4.13.
7. It is known that in a large city about one in twenty of the people are Muslim. People are stopped at random until a Muslim person is found.
 - a) Find the type of distribution, and hence write down the mean and variance of this distribution.
 - b) 150 surveyors around the city stop the people until they have found someone of the Muslim faith. Find the probability that the sample mean will be between 21 and 28.
8. A die is rolled until 3 fives are obtained.
 - a) Find a suitable model for this distribution and hence find the mean and variance of the distribution.
 - b) Thirty samples are taken from this distribution. Find the probability that the sample mean **does not** lie between 16.5 and 19.5.

