Sampling from the normal distribution — IB HL Stats Option

1. Find an unbiased estimate of the population mean and population variance from which these samples have been drawn.

   a) 5, 7, 8, 9, 10, 12, 15

   b) 101, 106, 97, 106, 108, 91, 87, 130

2. A population has a mean of 60 and a variance of 25. Find the expected mean and variance of the sample when the sample size is,

   a) 10 b) 20 c) 50

3. A random sample of 10 children's heights in a Year 1 class was calculated in a school and the mean and variance of the sample were calculated as 120 cm and 8 cm respectively.

   a) Use this information to find unbiased estimators of the population mean and the population variance.

   b) Use the sample to find the probability that the population mean lies between 110 cm and 130 cm.

   c) State any assumptions that have been made when calculating the answers to a) and b).

4. Fish in a pond have been sampled previously and their lengths are known to follow a normal distribution with a mean of 18 cm and a standard deviation of 2 cm.

   a) A sample of 20 fish are taken from the pond. Calculate the expected mean and variance of the fish in the sample.

   b) A further sample of 40 fish are taken from the pond. Calculate the expected mean and variance of the fish from this sample.

5. Cartons of juice are filled, with the volumes following a normal distribution. A sample of 25 cartons of juice is taken and the sample mean is found to be 990 ml and the sample variance is 3 ml.

   a) Assuming the sample was random, calculate unbiased estimators for the sample mean and variance based on sample.

   b) Calculate the probability that a carton of juice drawn from the population has a volume in excess of 1 litre.

   c) The manufactures claim that the cartons of juice contain 1 litre of juice. Do you think the claims are justified? Give reasons for your answers.

6. Kookabird make cricket balls that they claim have a mean of 400g and a variance of 9g that follows a normal distribution. A sample of 25 balls was taken.

   a) Calculate the expected mean and standard deviation (standard error) of the sample.

   b) In fact when the sample was taken the sample mean was 402g and the standard error of the sample mean was $\frac{3}{7}$.
   Calculate unbiased estimators for the mean and variance of the population based on the sample.

   c) Use your answer in b) to find the percentage of cricket balls that are within 1 standard deviation of the mean claimed by the manufacturer of the cricket balls.

   [Hint: use the figures for mean and standard deviation claimed by the manufacturer to find your probability interval.]

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ANSWERS

1. a) $\mu = 9.43$ $\sigma^2 = 65.5$

   b) $\mu = 103$ $\sigma^2 = 1211$

2. a) $\overline{x} = 60$ $S^2 = 2.5$

   b) $\overline{x} = 60$ $S^2 = 1.25$

   c) $\overline{x} = 60$ $S^2 = 0.5$

3. a) $\mu = 120$ $\sigma^2 = 80$

   b) Prob. = 0.737

   c) Population distribution was normal and the sample was random and fair.

4. a) $\overline{x} = 18$ $S^2 = \frac{1}{5}$

   b) $\overline{x} = 18$ $S^2 = \frac{1}{10}$

5. a) $\mu = 990$ $\sigma^2 = 75$

   b) Prob. = 0.124

   c) Claims are not justified as the probability of bottles at 1000 ml or above is too small.

6. a) $\overline{x} = 400$ $S = \frac{3}{5}$

   b) $\mu = 402$ $\sigma^2 = \frac{225}{49}$

   c) Prob. = 0.67