

1. A machine is known to dispense liquid cartons such that the volume dispensed is normally. It is claimed by the manufacturer of the machine that the mean volume dispensed is 100 ml.

In a series of trials on the machine, the following values were recorded.

104.4, 101.7, 103.6, 98.2, 92.2, 97.6, 96.2, 105.8, 94.7, 98.6

- Find a 95% confidence interval for the mean volume of liquid dispensed by the machine.
 - Use your interval to comment on the claim of the manufacturer.
2. Sugar is packed into bags, which are labeled as containing 1 kg. However, there have been complaints from Tesco supermarket that the bags do not contain 1 kg of sugar.

Statisticians at Tesco take a random sample of 8 packets of sugar from the shelves and weigh them. The results are below, with the mass given in grams.

1020 990 920 980 1010 1070 910 890

Tesco suggest carrying out a t -test at the 5% level of significance to test the mass of the bags of sugar.

- Justify the use of the t -test in this case.
- Write down suitable null and alternate hypothesis for this test.
- Calculate the sample mean and standard error for the sample.
- Write down the degrees of freedom (ν), and the critical t -value from the table.
- Find the test statistic.
- By using your results from above clearly state a conclusion.

t-tests

IB HL Stats Option

3. Students at Camberwick High School have just sat their mock examinations. The Principal reviewed the grades and was concerned that the Physics examination was too easy when compared to the maths examination.

In order to test the principal's claim the 10 students were chosen at random and their results, in percentages, have been displayed below.

	A	B	C	D	E	F	G	H	I	J
Maths	45	52	65	71	89	38	47	54	68	73
Physics	49	58	62	74	95	48	62	71	81	74

Carry out a paired t -test at the 1% level of significance to test the claim that the Physics examination was easier than the maths examination.

4. A fisherman caught 8 trout from a lake. He weighed each and wrote the masses in a table as follows. The masses are in grams.

794 904 532 470 630 500 781 587

The fisherman is writing an article for the world famous magazine 'What Trout?' and wants to state a 99% confidence interval for the mean mass of trout in the lake. Use a t -test to find this confidence interval for the fisherman.

5. Sam believes he watches about 3 hours of TV per day. To test this belief Sam makes a record of the amount of hours he has spent watching TV over a week as follows:

Sun	Mon	Tues	Wed	Thur	Fri	Sat
3.9	2.3	3.1	3.7	2.5	1.8	5.1

You may assume that the times are normally distributed.

- State suitable hypotheses.
- Test Sam's belief at the 5% significance level.
- Justify your choice of test.

6. A fisherman thinks that the masses of trout have decreased during the 6 month since his last sample. He tags the 8 fish he caught in the summer and then catches them again in the winter.

Their masses are shown below

Fish	A	B	C	D	E	F	G	H
Summer	794	904	532	470	630	500	781	587
Winter	610	790	401	390	540	320	790	460

- Write down suitable hypotheses for the test.
- Calculate the summer mass – winter mass for each fish.
- Calculate the estimate of the population variance of the (summer mass – winter mass).
- Find the critical value from the t tables, stating your degrees of freedom, ν , at the 95% level of significance.
- Calculate the test statistic.
- Use your answers from above to make a suitable conclusion at the 95% level of significance.

Answers

1. a) 96.02 – 101.98
 b) Claim is reasonable.
2. a) t-test can be used because the sample is small and the population variance is unknown.
 b) $H_0 : \mu = 1000$
 $H_1 : \mu \neq 1000$
 c) $\bar{x} = 973.75, s_{n-1} = 62.09, s.e = 21.95$
 d) $v=7$, critical value = 2.365
 e) -1.195
 f) Do not reject H_0 in favour of H_1 .

3. $H_0 : \mu_1 = \mu_2$
 $H_1 : \mu_1 < \mu_2$

$$\sum d = 72, \sum d^2 = 890$$

$$\hat{\sigma}^2 = \frac{890}{9} - \frac{72^2}{90} = 41.29$$

$$t_{test} = \frac{7.2 - 0}{\frac{6.43}{\sqrt{10}}} = 3.54$$

$v=9$, critical value is 2.821 from table.

Reject H_0 in favour of H_1 , there is sufficient evidence at the 99% level of significance to suggest that the Physics examination was easier than the mathematics examination.

t-tests

IB HL Stats Option

4. 517.31 – 782.19

5. a) $H_0 : \mu = 3$
 $H_1 : \mu \neq 3$

b) $\bar{x} = 3.2, s_{n-1} = 1.13, n = 7, \mu = 3$

$$t_{test} = \frac{3.2 - 3}{\frac{1.13}{\sqrt{7}}} = 0.468$$

$v=6$, critical value = 1.943

Do not reject H_0 in favour of H_1 .

c) Use the t-test because the sample is small and we do not know the population variance.

6. a) $H_0 : \mu_1 = \mu_2$
 $H_1 : \mu_1 > \mu_2$

b)

Fish	A	B	C	D	E	F	G	H
Difference	184	114	131	80	90	180	-9	127

c) 3792.41

d) 1.895, $v=7$

e) 5.15

f) Reject H_0 in favour of H_1 , there is sufficient evidence at the 95% level of significance to suggest that the mass of the fish in winter is less than the mass of the same fish in the summer.