

## Statistics and probability review questions

IB HL

1. A die is numbered as follows: 1,1,2,3,4,5. The die is rolled twice and the scores are added. A discrete random variable table is constructed.

a) Copy and complete the discrete random variable table.

$x$	2	3	4	5	6	7	8	9	10
$P(X=x)$	$\frac{4}{36}$			$\frac{6}{36}$					

- b) Find  $E(X)$  and  $V(X)$ .
- c) Rob and Sally play a game whereby Rob rolls the die twice and adds up the score. If Rob makes an even number he wins, otherwise Sally will roll the die twice. If Sally makes an even number then she wins. Rob and Sally continue to play like this.
- Calculate the probability that Rob will win on his go, if Rob goes first.
  - Calculate the probability that Rob wins if Sally goes first.
  - Given that Rob wins on his first go, calculate the probability that Rob went first rolling the dice.
  - Rob rolls the dice first. Calculate the probability that Rob wins the game.
2. Calculators at a factory are tested randomly to check if they are faulty. It is known from previous experience that the probability of a faulty calculator is known to be 0.04. If a faulty calculator is tested the probability of it testing faulty is 0.85. If a non-faulty calculator is tested the probability of it testing faulty is 0.06.

Given that the calculator was tested as not been faulty, calculate the probability that it was faulty.

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3. Phone calls to an insurance company occur randomly at a rate of 3 per hour during the mornings and at a rate of 2.5 per hour in the afternoon.
- Give two reasons why these distributions can be considered as a Poisson distribution.
  - Calls are monitored between 10 A.M. and 11 A.M. Calculate the probability that,
    - 0 calls are received,
    - at least 3 calls are received,
    - exactly 4 calls are received, given that at least 3 calls are received.
  - Calls are monitored between 10 A.M. and 11 A.M. and then again between 3 P.M. and 4 P.M.

During this time period calculate the probability that a total of 3 calls are received.

4.  $X$  is normally distributed with a mean of  $\mu$  and a standard deviation of  $\sigma$ .

Find the value of  $\mu$  and  $\sigma$ , given that  $P(X > 20.126) = 0.1$  and  $P(X < 10.854) = 0.15$ .

5. Events  $A$  and  $B$  are independent.  $P(A \cup B) = 0.42$  and  $P(A) = 0.4$ , find  $P(B)$ .
6. In a village the probability of someone having an illness is known to be  $\frac{1}{7}$ . A sample of 15 people are chosen and tested for the illness. Calculate the probability of,
- exactly 3 people in the sample have the illness,
  - at least 3 people in the sample having the illness.
  - exactly 3 people in the sample have the illness, given that at least 3 people in the sample have the illness.

7. The AMCE widget company produce widgets that are normally distributed with a mean weight of 25g and a standard deviation of 5g. Widgets that weigh less than 18g and more than 35g are considered to be substandard.
- Calculate the probability that a widget will be substandard.
  - Widgets cost \$2 to produce and are sold for \$5. Customers can return all substandard widgets and the money returned for each widget returned. However, it is found that only 50% of the customers return faulty widgets.

Calculate the profit the company expects to make in a month when 6000 widgets are sold.

8. A continuous probability distribution function is defined as,

$$f(x) = kx^2(2 - x) \text{ when } 0 \leq x \leq 2, \text{ otherwise } P(x) = 0.$$

- Show that  $k = \frac{3}{4}$ .
- Find  $E(X)$  and  $V(X)$ .
- Sketch the graph of  $f(x)$ , indicating clearly on the graph where the mode lies, and state the value of the mode.
- calculate the value of the median,  $m$ .
- Find  $P(1 \leq x \leq 1.5)$ .

**ANSWERS**

1. a)

$x$	2	3	4	5	6	7	8	9	10
$P(X=x)$	$\frac{4}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{7}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

b)  $E(X) = \frac{32}{6}$        $V(X) = \frac{40}{9}$

c) i)  $\frac{5}{9}$       ii)  $\frac{20}{81}$       iii)  $\frac{5}{9}$   
 iv)  $\frac{9}{13}$

2. 0.0066

3. a) Fixed time and the distribution is random.

b) i) 0.05      ii) 0.577      iii) 0.29

c) 0.113

4.  $\mu = 15, \sigma = 4$

5.  $P(B) = 0.3$

6. a) 0.209      b) 0.365      c) 0.574

7. a) 0.10351      b) \$16447.35

8. b)  $E(X) = 1.2$        $V(X) = 0.1575$

c) mode =  $\frac{4}{3}$

d) 1.23

e) 0.56