

Vectors and matrices test

IB SL

1. Given that $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{c} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$.

a) Evaluate $\mathbf{a} \cdot \mathbf{b}$

b) Evaluate $\mathbf{b} \cdot \mathbf{c}$

c) What can you conclude from your answer to a)? [3 marks]

2. Matrix A is defined as
$$\begin{bmatrix} 4 & -4 & 3 \\ 1 & 0 & 2 \\ -1 & 4 & 3 \end{bmatrix}.$$

Show that A is singular, i.e. it has no inverse. [3 marks]

3. Solve the set of simultaneous equations,

$$3x - y + z = 9$$

$$x + 2y - z = -7$$

$$5x + y - 3z = -9$$

[3 marks]

4. Find the vector equation of the straight line, in the form $\mathbf{r} = \mathbf{p} + t\mathbf{d}$, that joins the two points $A(4, 2)$ and $B(-5, 10)$. [3 marks]

5. $x = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $y = 5\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}$.

Find the angle between the two vectors x and y . [2 marks]

6. A triangle has the vertices $A(5,3)$, $B(3,2)$, and $C(5,1)$.

a) Write the vector AB in the form $\begin{pmatrix} x \\ y \end{pmatrix}$. [1 mark]

b) Write the vector AC in the form $\begin{pmatrix} x \\ y \end{pmatrix}$. [1 mark]

c) Find the angle BAC . [3 marks]

d) By finding the magnitudes of AB and AC , find the area of the triangle ABC . [3 marks]

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7. A ship is sailing at 26 km/h, with a starting point of $(-4, -10)$ and a direction of $5\mathbf{i} + 12\mathbf{j}$.

a) Show that the vector equation of the ship's path is,

$$r = \begin{pmatrix} -4 \\ -10 \end{pmatrix} + t \begin{pmatrix} 10 \\ 24 \end{pmatrix}, \text{ where } t \text{ is the time in hours. [2 marks]}$$

b) Write the equation of the ship's path in the form, $ax + by + c = 0$. [2 marks]

c) Find the position of the ship after 3 hours of travelling. [1 mark]

d) A harbour is located at the origin $(0,0)$. Find how many hours and minutes after starting its journey ship is at its closest distance from the harbour. [3 marks]

8. The Moscow metro map is to be drawn on a coordinate grid. For this purpose the station of *Novokuznetskaya* is taken as the origin. The station *Arbatskaya* has coordinate $(-12, 5)$ and the station *Prospekt Mira* has the coordinate $(7, 24)$.

a) The station of *Proletaskaya* has the coordinate $(8, -5)$. The station of *Proletaskaya* lies on the straight line, L , that connects it to *Novokuznetskaya*. Find the equation of the line L , in the form $ax + by + c = 0$. [3 marks]

b) A metro line is to be built that connects the stations of *Proletaskaya* and *Arbatskaya*.

i) Find the unit vector that connects *Proletaskaya* and *Arbatskaya*, PA .

ii) Give the vector equation of this metro line. [5 marks]

c) Two other stations lie on different metro lines. They are modeled by the vector equations below.

$$\text{Polyanka: } -2\mathbf{i} - 5\mathbf{j} \quad \text{and} \quad \text{Mitino: } -3\mathbf{i} + 10\mathbf{j}$$

Find the obtuse angle created when the two vector lines intersect each other on the map. [2 marks]

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Answers

1. a) 0 b) 17 c) The vectors are perpendicular.

3. $x = 1, y = -2, z = 4$

4. $r = \begin{pmatrix} 4 \\ 2 \end{pmatrix} + t \begin{pmatrix} -9 \\ 8 \end{pmatrix}$ or $r = \begin{pmatrix} -5 \\ 10 \end{pmatrix} + t \begin{pmatrix} 9 \\ -8 \end{pmatrix}$

5. 96.7°

6. a) $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ b) $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ c) 63.4° d) 2 units^2

7. b) $12x - 5y - 2 = 0$

c) $(26, 62)$

d) 24 mins, 0.16 km

8. a) $5x + 8y = 0$

b) i) $\begin{pmatrix} 1 \\ \frac{1}{\sqrt{5}} \\ 2 \\ \frac{1}{\sqrt{5}} \end{pmatrix}$ ii) $r = \begin{pmatrix} 8 \\ -5 \end{pmatrix} + t \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

c) 144.9° .