

Vectors and matrices revision sheet

IB HL

1. Given that $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$ and $\mathbf{c} = \mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$, find $(\mathbf{a} \times \mathbf{c}) \cdot \mathbf{b}$.

2. $x = \begin{bmatrix} 4 & 0 \\ 5 & 2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find the value of λ such that $(X - \lambda I)$ is a singular matrix.

3. $A = \begin{bmatrix} 3 & 1 & -5 \\ 2 & 4 & -1 \\ -1 & -3 & 9 \end{bmatrix}$, $B = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $C = \begin{bmatrix} 17 \\ 20 \\ -31 \end{bmatrix}$.

a) Show that the determinant of A is 92.

b) Given that $AB = C$, find the values of x , y , and z .

4. Find the equation of the plane in the form $ax + by + cz = d$, that is perpendicular to the vector $\begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ and passes through $(2, -4, 5)$.

5. Find the equation of the plane in the form $ax + by + cz = d$, that contains the vectors $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix}$.

6. Find the area of the triangle that has vertices $(2, 1, 1)$, $(4, 2, 1)$, and $(-3, 2, -1)$.

7. Find the angle between the plane $3x - 2y + 4z = 4$ and the y -axis. Give your answer to the nearest degree.

8. a) Show that the following set of equations have no solutions,

$$2x + y - 3z = 4$$

$$4x + y + 4z = 2$$

$$2x + 5y - 43z = 30$$

b) Find the unique set of solutions for the following set of simultaneous equations.

$$3x + 5y + z = 0$$

$$2x - y + 8z = 3$$

$$x + 10y - z = 7$$

9. Find the normal vector to the plane that contains the **a** and **b** as defined as follows:

$$\mathbf{a} = 3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \mathbf{b} = \mathbf{i} - \mathbf{j} - 5\mathbf{k}.$$

10. Find the vector equation that is formed at the intersection of the planes:

$$\pi_1 : 3x - y + z = -2 \quad \text{and} \quad \pi_2 : 2x + y - 2z = 1$$

11. Show that the lines $\frac{x-1}{1} = \frac{y-3}{5} = \frac{z-4}{1}$ and $\frac{x-3}{1} = \frac{y-10}{2} = \frac{z-6}{1}$ intersect and find the coordinate at the point of intersection.

12. In this question $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ represents a displacement of 1 km north and $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ represents a displacement of 1 km east.

Two ships set sail from a port that has the coordinates $P(20,35)$.

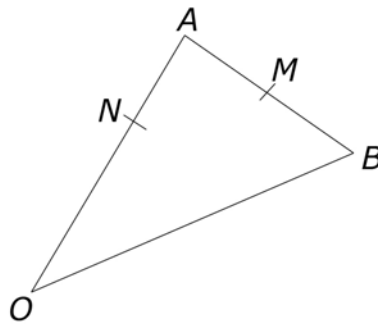
Ship *X* sails on a straight line for one hour to the coordinate $(50,75)$.

Ship *Y* sails on a straight line for one hour to the coordinate $(-35,10)$.

- Find the average speed of ship *X* in km/h.
- Give the bearing that ship *X* is sailing on.
- Find the vector equation of the straight line that ship *Y* is sailing on.
- Find the obtuse angle between the two ships after one hour.
- Find how far apart the ships are after one hour.

13. In the diagram below $N = \frac{2}{3}OA$ and $M = \frac{1}{2}AB$.

OA is represented by vector \mathbf{a} , and OB is represented by vector \mathbf{b} .



Find in terms of \mathbf{a} and \mathbf{b} ,

- a) AB
- b) AM
- c) OM
- d) NM

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Answers:

1. 53

2. $\lambda = 2, \lambda = 4$

3. b) $x = 1, y = 4, z = -2$

4. $3x - 2y + z = 19$

5. $2x + 15y - 8z = 0$

6. Area = 4.15 units²

7. 111.8°

8. b) $x = -2, y = 1, z = 1$

9. $\begin{pmatrix} 22 \\ 17 \\ 1 \end{pmatrix}$ or $22i + 17j + k$

10. $\frac{5x+1}{1} = \frac{5y-13}{8} = \frac{z}{1}$

11. Prove that $\lambda = 1$ and $\mu = -1$ or equivalent. Coordinate is (2,8,5).

12. a) 50 km/h

b) 037°

c) $r = \begin{pmatrix} 20 \\ 35 \end{pmatrix} + t \begin{pmatrix} -55 \\ -25 \end{pmatrix}$ or $r = \begin{pmatrix} 20 \\ 35 \end{pmatrix} + t \begin{pmatrix} -11 \\ -5 \end{pmatrix}$

d) 15

e) 106.9 km

13. a) $b - a$

b) $\frac{1}{2}b - \frac{1}{2}a$

c) $\frac{1}{2}a + \frac{1}{2}b$

d) $\frac{1}{2}b - \frac{1}{6}a$