

Vectors and matrices test

IB HL

1. Given that $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{c} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, find $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$. [4 marks]

2. A triangle has vertices $X(2,4,5)$, $Y(-1,3,2)$ and $Z(-3,0,1)$.

Find the angle YXZ . [4 marks]

3. a) Find the point of intersection of the three planes,

$$3x - y + z = 9$$

$$x + 2y - z = -7$$

$$5x + y - 3z = -9$$

[3 marks]

- b) A fourth plane with the equation $4x + 2y + 5z = d$ passes through the point of intersection found in a).

Find the value of d . [2 marks]

4. a) Find the value of k in the following set of equations such that the equations have an infinite set of solutions.

$$2x + y + 4z = k$$

$$2x + 5y + 7z = 3$$

$$x - 3.5y - z = 0$$

[5 marks]

- b) For this value of k find the general set of solutions for these equations. [3 marks]

5. $\mathbf{x} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{z}$ and $\mathbf{y} = 5\mathbf{i} + 4\mathbf{j} - 7\mathbf{z}$.

Find the angle between the two vectors \mathbf{x} and \mathbf{y} . [2 marks]

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6. A ship is sailing at 26 km/h, with a starting point of $(-4, -10)$ and a direction of $5\mathbf{i} + 12\mathbf{j}$.

- a) Show that the vector equation of the ship's path is,

$$r = \begin{pmatrix} -4 \\ -10 \end{pmatrix} + t \begin{pmatrix} 10 \\ 24 \end{pmatrix}, \text{ where } t \text{ is the time in hours.} \quad [2 \text{ marks}]$$

- b) Write the equation of the ship's path in the form,

$$ax + by + c = 0. \quad [2 \text{ marks}]$$

- c) Find the position of the ship after 3 hours of travelling. [1 mark]

- d) A harbour is located at the origin $(0,0)$. Find the closest distance from the ship to the harbour. [2 marks]

7. A point X is at the foot of the perpendicular from the point $(2, 1, 6)$ to the plane $3x + y - z = 16$.

Find the coordinates of X . [4 marks]

8. Two planes in space have the equations:

$$\pi_1: 2x + 3y + z = 6$$

$$\pi_2: 6x - y - z = -2$$

The point P has coordinates $(1, -2, 10)$.

- a) Show that P lies in both planes. [2 marks]

- b) Find the vector equation of line where the two planes intersect. [4 marks]

- c) Find the angle between the two planes. [3 marks]

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Answers

1. -21

2. 19.6°

3. a) $x = 1, y = -2, z = 4$

b) $d = 20$

4. a) $k = 2$

b) $x = \frac{7 - 13t}{8}, y = \frac{1 - 3t}{4}, z = t$

5. 96.7°

6. b) $12x - 5y - 2 = 0$

c) (26, 62)

d) 1 hour 39 minutes

7. $\lambda = 3$ and coordinate (11, 4, 3).

8. b)
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 8 \\ -20 \end{pmatrix}$$

c) 69.7°