***IB SL – Logarithms and exponentials***

## Laws of logarithms

$$\log_b a = \frac{\log b}{\log a}$$

$$\log x + \log y = \log xy$$

$$\log x - \log y = \log \frac{x}{y}$$

$$\log x^n = n \log x$$

**A log can be to any base, your calculator works in base 10 (log) and base e, the natural logarithm (ln).**

Examples of using the formula.

1. Write the following expression as a single logarithm.

$$2\log_a 3 - 4\log_a 2 + \log_a 32$$

Answer: $\log_a 9 - \log_a 16 + \log_a 32$

$$\log_a \frac{9 \times 32}{16}$$

$$\log_a 18$$

2. $x = \log_5 2$ and $y = \log_5 9$, express in terms of $x$ and $y$,

$$\log_2 9$$

Answer: Using the change of base rule we can see that we have,

$$\log_2 9 = \frac{\log_5 2}{\log_5 9}$$

$$\frac{\log_5 2}{\log_5 9} = \frac{x}{y}$$

## Applications of logarithms

1. Logarithms can be used to solve problems where the missing number is an index, such as,

   $3^{x+1} = 18$

   $\ln(3^{x+1}) = \ln(18)$

   $x + 1 = \dfrac{\ln 18}{\ln 3}$

   $x = 2.63 - 1$

   $x = 1.63$

2. Logarithms are also used to solve missing numbers in growth functions that are expanding exponentially.

   Example:

   The amount of bacteria in a biology experiment is modelled by the function:

   $P_t = 5e^{kt}$, where $t$ is the time in hours and $P$ is the amount of bacteria measured in grams.

   (i) Find the initial amount of bacteria.

   *The initial value t=0.*

   $P_0 = 5e^0$
   $P_0 = 5$

   (ii) After 5 hours there is 9.34123g of bacteria present.
   Show that the value of $k$ = 0.125 correct to 3 decimal places.

   $9.34123 = 5e^{5k}$

   $\dfrac{9.34123}{5} = e^{5k}$

   $\ln(\dfrac{9.34123}{5}) = 5k$

   $k = \dfrac{0.625}{5}$

   $k = 0.125$

(iii) Calculate after how many hours will the bacteria reach 12g.

$12 = 5e^{0.125t}$

$\ln(\frac{12}{5}) = 0.125t$

$t = \frac{0.8755}{0.125}$

$t = 7$