

Laws of logarithms

$$\log_b a = \frac{\log a}{\log b}$$

$$\log x + \log y = \log xy$$

$$\log x - \log y = \log \frac{x}{y}$$

$$\log x^n = n \log x$$

A log can be to any base, your calculator works in base 10 (log) and base e, the natural logarithm (ln).

Examples of using the formula.

1. Write the following expression as a single logarithm.

$$2\log_a 3 - 4\log_a 2 + \log_a 32$$

Answer: $\log_a 9 - \log_a 16 + \log_a 32$

$$\log_a \frac{9 \times 32}{16}$$

$$\log_a 18$$

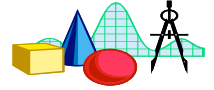
2. $x = \log_5 2$ and $y = \log_5 9$, express in terms of x and y ,

$$\log_2 9$$

Answer: Using the change of base rule we can see that we have,

$$\log_2 9 = \frac{\log_5 9}{\log_5 2}$$

$$\frac{\log_5 9}{\log_5 2} = \frac{y}{x}$$



Applications of logarithms

1. Logarithms can be used to solve problems where the missing number is an index, such as,

$$3^{x+1} = 18$$

$$\ln(3^{x+1}) = \ln(18)$$

$$x+1 = \frac{\ln 18}{\ln 3}$$

$$x = 2.63 - 1$$

$$x = 1.63$$

2. Logarithms are also used to solve missing numbers in growth functions that are expanding exponentially.

Example:

The amount of bacteria in a biology experiment is modelled by the function:

$P_t = 5e^{kt}$, where t is the time in hours and P is the amount of bacteria measured in grams.

- (i) Find the initial amount of bacteria.

The initial value $t=0$.

$$P_0 = 5e^0$$

$$P_0 = 5$$

- (ii) After 5 hours there is 9.34123g of bacteria present.
Show that the value of $k = 0.125$ correct to 3 decimal places.

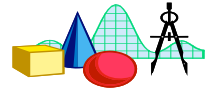
$$9.34123 = 5e^{5k}$$

$$\frac{9.34123}{5} = e^{5k}$$

$$\ln\left(\frac{9.34123}{5}\right) = 5k$$

$$k = \frac{0.625}{5}$$

$$k = 0.125$$



(iii) Calculate after how many hours will the bacteria reach 12g.

$$12 = 5e^{0.125t}$$

$$\ln\left(\frac{12}{5}\right) = 0.125t$$

$$t = \frac{0.8755}{0.125}$$

$$t = 7$$