Sequences and series

At IB level you need to know about two types of series: *arithmetic sequence* and *geometric progression.* For each one you must be able to calculate the next term and find the sum of terms.

What is the difference between the two?
An *arithmetic sequence* is one where the numbers go up (or down) by adding (or subtracting) by the same number each time, known as the *common difference*.
A geometric sequence is one where the numbers go up (or down) by multiplying by the same number each time, known of the *common ratio*.

Sequence	Type
6, 9, 12, 15, 18, ……	Arithmetic
1, 3, 9, 27, 81, ……	Geometric
25, 20, 15, 10, 5, ……	Arithmetic
$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$, ……	Geometric

The first problem with these questions is to establish if we have an AP or a GP. Listing out the terms given should help you with this.

Using the formula sheet

The nth term of an arithmetic sequence: $u_n = u_1 + (n-1)d$

The sum of n terms of an arithmetic sequence: $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$

The nth term of a geometric sequence: $u_n = u_1 r^{n-1}$

The sum of n terms of a geometric sequence: $S_n = \frac{u_1(r^n - 1)}{r-1} = \frac{u_1(1-r^n)}{1-r}$, $r \neq 1$

The notation
u_n = the next term of a sequence
S_n = the sum of a sequence
a = the first number of a sequence
d = the common difference for an arithmetic sequence
l = the last term of a sequence
r = the common ratio of a geometric sequence
n = the number of terms

The Summation sign

The Greek capital letter $\sum$ (sigma) is used to indicate the sum of a series.

The sign is used with various numbers to find sum of sequences. Substitute numbers into the sequence to identify the first term and the common ratio or the common difference.

This is shown below in the table.

Sequence	**First 3 terms**	**First term / Common difference or ratio / n**	**Answer**
$\sum_{r=1}^{r=10} 3r - 1$	2, 5, 8	*AP:* $U_1=2$ $d=3$ n=10	155
$\sum_{r=1}^{15} 1.05^r$	1.05, 1.1025, 1.1576	GP: $U_1=1.05$ $r=1.05$ n=15	26.39
$500\sum_{r=5}^{10} 1.10^r$	1.1, 1.21, 1.331	GP $U_1=1.1$ $r=1.1$ n=6	500 x 8.487171 = 4243.59

The sum to infinity of a geometric series

Taking the formula for the sum of a geometric series: $S_n = \frac{u_1(1-r^n)}{1-r}$.

If the common ratio is less between -1 and 1, e.g. a fraction then as *n* becomes large, so the sum of the series will tend towards one number.

This is because if you take the bracket $(1-r^n)$, *r* is a fraction less than 1 and *n* is large so the bracket becomes (1 – 0), so the formula of a sum to infinity as: $\frac{u_1}{1-r}$.

Guided example

Devon has a savings scheme. He starts off by adding \$10, the next month he adds \$12, the following month \$14 and so on.

(a) Calculate how much Devon has adds on the 14th month.

(b) Calculate how much Devon has saved after 2 years.

Answer (a)

- Firstly, establish if it is an AP or a GP by listing out the sequence: 10, 12, 14, 16, 18, etc.
- As the same number is being added on each time the sequence is an AP.
- Find the numbers to go into the formula: a = 10, d = 2, n = 14 (as you are finding the 14th month).
- Now put all these numbers into the formula u_n = 10 + (13 x 2).
- The answer is \$36.

Answer (b)

- We have already established that it is an AP, and we know all the letters' values.
- Use the formula for the sum of a formula: S_n = 24/2 (2x10 + (23x2)), the answer will be \$792.

Guided example

A swimmer is training for a long distance race. She will start on the first day by swimming 50m, and each day increase the number of metres she swims by 10% for the first 30 days.

(a) Calculate how far the swimmer must swim on the 30th day.

(b) Calculate how many metres the swimmer swum in 30 days.

Answer (a)

- Like the question above start by listing the sequence: 50, 55, 60.5, 66.55, etc
- As the sequence is being multiplied each time it is a GP.
- Find the numbers to go into the formula: a = 50, r = 1.1, n = 30.
- Put these numbers into the formula u_n = 50 x 1.1^{30}.
- The answer is 872 metres.

Answer (b)

- We have already established that this is a GP, and we have the letters' values.
- Use the formula for S_n = [50 x (1.1^{30} – 1)] ÷ [1.1 – 1], the answer is 8225 metres.